A Thermodynamic-Combinatorial Approach to the Design of Optimum Heat Exchanger Networks

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This article describes a novel approach to the systematic synthesis of heat exchanger networks. For a given synthesis problem, this approach leads to a complete listing of all solutions which exist, using a prescribed degree of energy recovery, the minimum number of exchangers, heaters and coolers, and no split streams. The approach is combinatorial, with a variety of thermodynamic criteria used to minimize the problem size by preventing infeasible solutions being generated. Two examples (5SP1 and 6SP1) are discussed to illustrate complete solution by hand.

SCOPE

Systematic methods for the preliminary design of heat exchanger networks have recently been described by Hohmann and Lockhart (1976), Nishida et al. (1977), and Linnhoff and Flower (1978). Most of these do not guarantee the global optimum cost network as a result, but merely produce a number of good approximations. The final selection is made from the list of networks generated, based on criteria such as reliability, start-up, flexibility in operation, safety, etc. There is a need for better ways to generate a relevant short list of "minimum cost" networks from which the candidate structures can be selected for more detailed design and evaluation. A relevant short list may be said to exclude *all* unsuitable structures while leaving out *no* promising structure.

Since the total cost of a network can be considered in two parts, i.e., operating costs and capital charges, recent work has concentrated on finding networks which achieve maximum energy recovery while showing low capital costs. In turn, low capital costs tend to be associated with the smallest number of units (i.e., heaters, coolers, and exchangers).

In our method, searches may be carried out to generate all solutions for a given problem with a prescribed degree of energy recovery, the minimum number of units, and no split streams. Specifying the prescribed degree of energy recovery as maximum using techniques described by Linnhoff and Flower (1978), the approach leads to an exhaustive list of the most attractive networks (if such solutions exist).

The method is based on combinatorial principles different from tree searching methods. It uses thermodynamic and topological arguments to reduce the size of the combinatorial problem. The method is called the TC method (for "thermodynamic/combinatorial") and appears to be equally well suited for fully automatic application in a computer program as for implementation by hand. If used by hand, problem-individual arguments may be formulated to eliminate the combinatorial problem almost completely. If used in the form of a computer program, generally applicable (and therefore simpler) arguments are employed. The method is described by solving two examples (5SP1 and 6SP1) for which maximum energy recovery/minimum number of units solutions exist. The prospects of the method are then discussed in the context of problems for which such solutions do not exist (i.e., problems in which maximum heat recovery would be more difficult to achieve).

CONCLUSIONS AND SIGNIFICANCE

The TC method as described permits listing of all solutions for a particular problem subject to the following conditions: 1) the networks show a prescribed e.g. minimum consumption of utilities; 2) the networks use the minimum number of units; and 3) the networks do not use stream splitting.

The combinatorial problem usually thought to be associated with the synthesis of heat recovery networks is kept to manageable size by a variety of topological and thermodynamic arguments. As demonstrated, problems like 5SP1 and 6SP1 can thus be solved exhaustively by hand with very little effort. When implemented on the computer, the TC method would appear to be of a greater potential than any other combinatorial method presented previously. An apparent disadvantage of the method is the fact that where solutions of the kind specified above do not exist, the method will yield no answer. This situation would be encountered in problems where full heat recovery is difficult to achieve, i.e., where more than the minimum number of units have to be used and/or streams have to be split for a satisfactory solution. However, partial solution of such problems is rapid by means of existing techniques (Linnhoff and Flower 1978), and the remaining problem is amenable to solution by the TC method.

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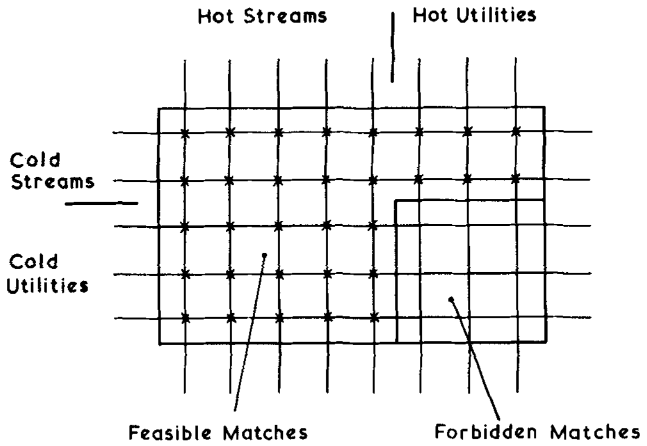


Figure 1. Identifying the feasible matches in a problem.

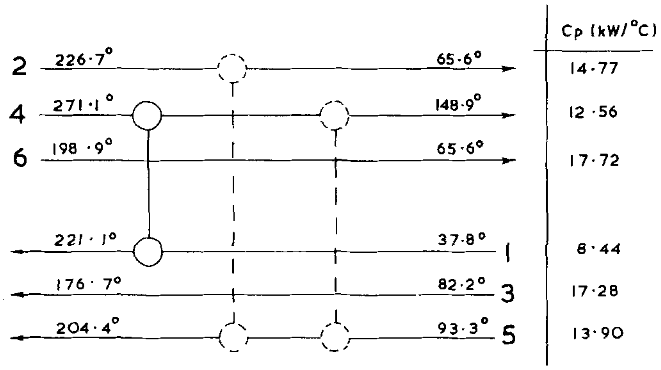


Figure 2. Identifying compulsory matches for Problem 6SP1.

THEORETICAL BASIS

Consider a problem with n_h hot streams and n_c cold streams, each of specified heat capacity flowrate, supply and target temperatures. The minimum requirement for heating and cooling services can be identified prior to network design by using the problem table described by Linnhoff and Flower (1978). The minimum number of heaters, coolers, and exchangers required to solve the problem, $n_{\min}$, is given by Hohmann (1971) as one less than the total number of streams and services

$$n_{\min} = n_h + n_c + n_{hs} + n_{cs} - 1 \qquad (1)$$

with n_{hs} = number of hot services, and
n_{cs} = number of cold services

Although this number may not be attainable in certain circumstances (Linnhoff 1979), it usually represents a feasible target in simplification of networks. In exceptional circumstances, the heat load of one of the exchangers may be identically zero, yielding an even simpler network. Similarly, important gains are to be made in practice by neglecting very small exchangers subject to slight relaxation of problem data (adjustments of stream supply and/or target temperatures, discussed later).

Apart from the minimum number of units, the number of different possible matches in a problem, n_{poss}, may be evaluated as demonstrated in Figure 1 and expressed

$$n_{\text{poss}} = (n_h + n_{hs}) \times (n_c + n_{cs}) - (n_{hs} \times n_{cs}) \qquad (2)$$

With this information, we can tackle the simple combinatorial problem of finding all possible sets of $n_{\min}$ matches that can be formed when selecting from n_{poss} different ones:

$$C_{n_{\text{poss}}}^{n_{\min}} = \frac{n_{\text{poss}}!}{n_{\min}! \times (n_{\text{poss}} - n_{\min})!} \qquad (3)$$

The number of different sets that fulfill this description is easily determined. Each set represents a selection of matches (specified by reference to the hot and the cold stream) that may form a maximum energy recovery/minimum number of units solution.

However, those sets of matches that will produce feasible networks must satisfy a number of constraints arising from topological and thermodynamic considerations. It proves possible to employ many of these constraints in a fundamental logical manner, so that most of the infeasible sets need never be generated. This approach is taken since, however efficiently checking procedures may be programmed, it is preferable to avoid generating infeasible sets of matches altogether. Some of the arguments employed are briefly outlined below.

Target Temperature Feasibility

Each match bringing a stream to its target temperature must be with a stream or service whose supply temperature is compatible with that target temperature.

Topological Feasibility

Each stream or service must be used in at least one match. This condition can be used to determine a number of sets smaller than that indicated by Equation (3), but the formula obtained is rather complex. The problem of size may be reduced by many orders of magnitude for large problems, especially if there is a marked difference in the number of hot and cold streams (see Linnhoff 1979). In the case of 10 SP1 (for data, see Pho and Lapidus 1973), for example, Linnhoff (1979) obtained approximately 3.0×10^6 alternative sets of matches once that the topological feasibility condition is enforced. This compares favorably with the 10^{25} evaluations said to be required to guarantee optimality by means of a tree searching algorithm (Ponton and Donaldson 1974).

Heat Load Feasibility

> If a stream or service is matched only once, its partner must have an equal or larger heat load. A natural consequence of this is that in any feasible network, the stream or service with the largest heat load must have at least two matches. Further, the second largest stream or service must have at least two matches, unless matched against the largest stream or service etc.

These constraints are necessary conditions for feasibility, but not sufficient in themselves. A complete check on heat loads must show that all matches have non-negative loads, while compliance with the target temperature tests in no way eliminates the need for tests on temperature crossovers within matches.

However, intelligent use of these three preliminary feasibility checks helps to select only those sets of matches which are not obvious failures. It provides a drastic reduction in the amount of detailed checking necessary later on.

These conditions are, of course, well known and form the basis of several existing methods of synthesis. For example, the method of Ponton and Donaldson (1974) can be considered to employ the target temperature condition in choosing each match, the target temperature condition as an upper bound in sizing each match, and the load condition in closing the set of matches.

Implementation of these constraints in a computer program is considered easy, but to gain maximum advantage it is necessary to apply the constraints in a rather flexible manner to achieve maximum reduction in the number of networks requiring detailed rating. To illustrate these points, problems 6SP1 and 5SP1 will be discussed in detail.

PROBLEM 6SP1: A SIMPLE AUTOMATIC APPROACH

The data are as given by Pho and Lapidus (1973), and most are evident from Figure 2. Assuming, as usual, a minimum approach temperature of $\Delta T_{min} = 11.11°C$ (20°F), the minimum services required are 1) no steam and 2) cooling water to the extent of 1553.4 kW (530.1 $\times$ 10^4 Btu/hr) (see Linnhoff and Flower 1978). With this information, it is possible to calculate the minimum number of units by using Equation (1):

$$n_{min} = 3 + 3 + 0 + 1 - 1 = 6$$

and the number of possible matches that could exist in the problem by Equation (2):

$$n_{poss} = (3 + 0) \times (3 + 1) - (0 \times 1) = 12$$

The essential task of the TC method is to identify and to examine all possibilities there are for n_{min} matches out of n_{poss} possible ones. Equation (3) indicates that there are

$$\frac{12!}{6! \times 6!} = 924$$

such sets of six matches. A simple automatic synthesis algorithm might identify these 924 sets, one by one, and then proceed to compute the heat loads of the individual exchangers in each set. The heat load of any match in a minimum number of units solution is uniquely determined by the problem data (see Hohmann 1971). Thereby, many sets will be found to be infeasible: Experience based on tests with the simple three feasibility criteria outlined above suggests that less than 10% of the original number of sets indicated by Equation (3) will normally be found to be feasible.

Then a simple automatic procedure could start by arranging, for any of the remaining sets, the individual matches in any feasible sequence. Consider Figure 3. A set of six matches is shown which may represent a feasible selection of heat loads. The task of identifying all networks represented by this particular set is carried out in a systematic way by: 1) evaluating all sequences of connections to each individual stream, and 2) examining all combinations of the sequences so found on thermodynamic grounds (i.e., by checking temperatures).

The number of possible sequences of the connections to a particular stream is, according to simple mathematical theory, given by the number of permutations of the connections. Thus, there are 3! possible sequences for the connections to stream No. 2 in Figure 3, 1! for stream No. 4, 2! for stream No. 6 and so on. By multiplying these figures, the number of combinations described under point 2) above is obtained:

$$(3!) \times (1!) \times (2!) \times (2!) \times (1!) \times (1!) = 24$$

Usually, other sets of matches found to be feasible on grounds of heat load compatibility would lead to similar results, so that a simple automatic search algorithm of the kind described here would, in the case of 6SP1, have to evaluate a number of networks approximately equal to

$$924 \times \frac{1}{10} \times 24 = 2{,}220$$

to produce an *exhaustive* list of solutions that feature maximum energy recovery as well as the minimum number of units!

This figure, however, can be reduced even further by means of thermodynamic arguments when applied before, rather than after, determining the parameters n_{min} and n_{poss}. One such possibility is comparing all supply and target temperatures of streams and services. In the case of 6SP1 where no hot utility usage is predicted, it is obvious that there must be a match between streams No. 4 (hot) and No. 1 (cold). Otherwise, stream No. 1 could not reach its target temperature (see Figure 2). Also, there must be a match between either stream No. 2 or stream No. 4 (hot) and stream No. 5 (cold), since otherwise stream No. 5 could not reach its target temperature.

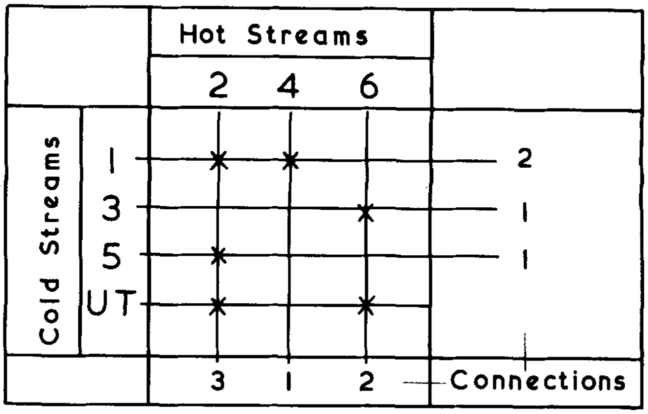


Figure 3. Number of connections on each stream for a feasible selection of matches in Problem 6SP1.

Another way of reducing the combinatorial problem is by examining the ways in which heaters and coolers may be arranged. The total heat load to be exchanged between process and utility streams is known from predicted utility requirements; it is obvious that limits must exist as to where in the network the heaters and coolers in question may be placed. In the case of 6SP1, only cooling is required. It is worth checking whether or not maximum energy recovery would be feasible with only one cooler: The answer may turn out negative. With only one cooler, heat may have to be removed from the process stream(s) at a higher average temperature than in the case of two coolers, so that less heat remains available for interprocess stream heat exchange at intermediate temperature levels. To check this, utility requirements can be examined for the modified problem, with the target temperatures of the hot streams adjusted to allow for the existence of single coolers situated at the cold ends of individual process streams. Modified in this way, the problem will require zero utility usage (if maximum energy recovery is still feasible), but produce a need for additional heating and cooling, to equal extents, if maximum energy recovery is no longer feasible.

As is easily predicted from the particular data for 6SP1, the least severe effect on the overall heat recovery situation results when a single cooler is situated on stream No. 6, since it has the largest Cp and the lowest target temperature. However, even here, maximum energy recovery is no longer feasible, an additional 35.2kW being required. Thus, any solution featuring no more than one cooler can be ruled out.

Next, the procedure may be repeated assuming two coolers. In this case, maximum energy recovery is found possible if the two coolers are situated on streams No. 2 and No. 6. If they were situated on streams No. 4 and No. 6, however, energy recovery would be impaired. In the case of streams No. 2 and No. 4, the deterioration in energy consumption would be even more severe. Finally, it is possible to assume three coolers, i.e., one for each hot stream, without affecting maximum energy recovery.

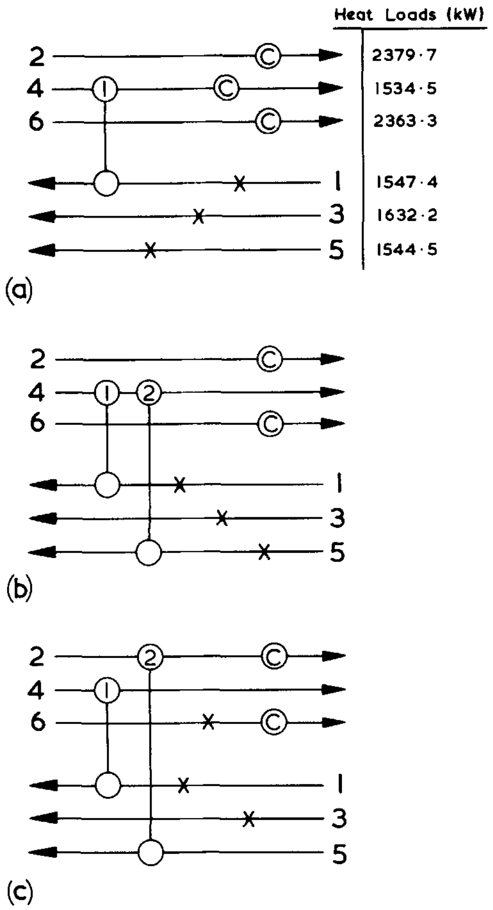


Figure 4. Initial arrangements for Problem 6SP1.

Using these simple findings, the combinatorial problem becomes much smaller: there are only two possibilities of choosing two of the matches between process streams (see Figure 2), and only two possibilities for selecting coolers. Thus, there are no more than four basic combinations on which further choices of matches may be based. For two of these combinations, only one further match may be chosen to bring the total up to six (these are two sets involving three coolers). For the other two combinations, two further matches may be chosen. With only seven possible matches to choose from in all cases, the number of different selections of six matches eventually obtained is 56:

$$2 \times C_7{}^1 + 2 \times C_7{}^2 = 56$$

Thus, 56 sets of six matches each (instead of the original 924) would be identified initially, and then checked on grounds of heat load compatibility, etc., in the course of the still rather simple search algorithm which includes these preliminary thermodynamic tests prior to the definition of the overall combinatorial problem. This reduction in number is quite dramatic, when compared with the figure of approximately 3.6×10^5 different networks that would be encountered in a conventional tree searching algorithm carried out for 6SP1 (estimated on the simple formula $(n_h \times n_c)!$ as given by Ponton and Donaldson, 1974). Perhaps even more surprising, however, is the ease with which an exhaustive search can be accomplished by hand calculation when using the TC method. Examples of this are presented below.

EXHAUSTIVE SEARCH BY HAND CALCULATION

Suppose that problem 6SP1 is to be solved by hand. The calculation of minimum energy requirements is a

TABLE 1. POSSIBILITIES FOR LAST TWO MATCHES IN 6SP1

No.	Match No. 3	Match No. 4	Discussion
1	6/1	2/3	see Fig. 5a
2	6/1	4/3	see Fig. 5b
3	6/1	6/3	see Fig. 5c
4	2/1	6/3	see Fig. 5d
5	4/1	6/3	Cyclic
6	6/1	6/3	see No. 3

simple exercise (Linnhoff and Flower 1978). It is assumed that the information is available that either three or two coolers (one on stream No. 2 and one on stream No. 6) are required. It is also evident that a match is required between streams No. 4 and 1 and another match either between streams No. 2 and No. 5, or between streams No. 4 and No. 5. Thus, the sketches shown in Figure 4 can be produced, in which the resulting initial selections of matches are presented. Any maximum energy recovery/minimum number of units solution for 6SP1 must be based on one of these three arrangements.

The purpose of the sketches is to facilitate the formulation of thermodynamic arguments by which it can then be confirmed whether or not a solution may result if any of these three arrangements are completed. This approach helps to considerably reduce the combinatorial problem that has to be overcome when identifying the last matches.

Consider Figure 4a. In this arrangement, the need for a further match for stream No. 1 is easily seen, since the heat load of stream No. 4 is not large enough to satisfy that of stream No. 1 in full. This situation, i.e., that there is a connection yet to be provided for stream No. 1, is indicated by means of a cross on stream No. 1 in Figure 4a. It is also evident that at least one connection must be made to stream No. 3, as well as to stream No. 5. (thus the crosses on streams No. 3 and No. 5). With three connections still required to three different cold streams, however, and only two more matches to choose (if the total is not to exceed six matches), a solution is not feasible. Therefore, it can be concluded that no maximum energy recovery/minimum number of units solution exists for 6SP1—which would be based on the arrangement shown in Figure 4a.

Next, consider Figure 4b. Here arguments similar to those above lead to the conclusion that at least one further connection is required for stream No. 1 and for stream No. 3. Then another connection must be made to stream No. 5 for the same reason as for stream No. 1: the heat load of stream No. 4 is too small. Thus, a similar situation is encountered as before: with three cold connections still required but only two more matches to choose, a solution is not feasible.

Finally, consider Figure 4c: Since the cooler on stream No. 6 cannot have a heat load larger than 1553.4 kW (530.1×10^4 Btu/hr) (i.e., the total need for cooling), at least one further connection will be required. Also, at least one other connection must be specified for stream No. 1 (on the same grounds as before), and one for stream No. 3. The remaining streams do not appear to obviously require further connections. Thus, there are only one hot and two cold connections still to be made and, with two matches yet to be chosen, feasible solutions might exist.

The problem is, by now, small enough to suggest further examination by combinatorial search. There are

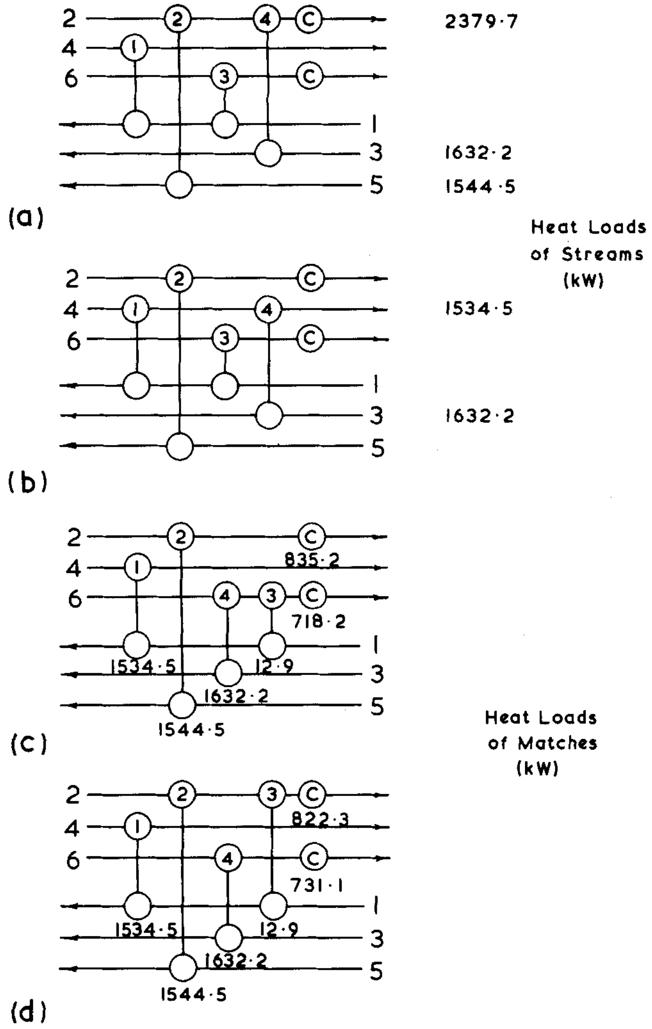


Figure 5. Finding two feasible sets of six matches for Problem 6SP1.

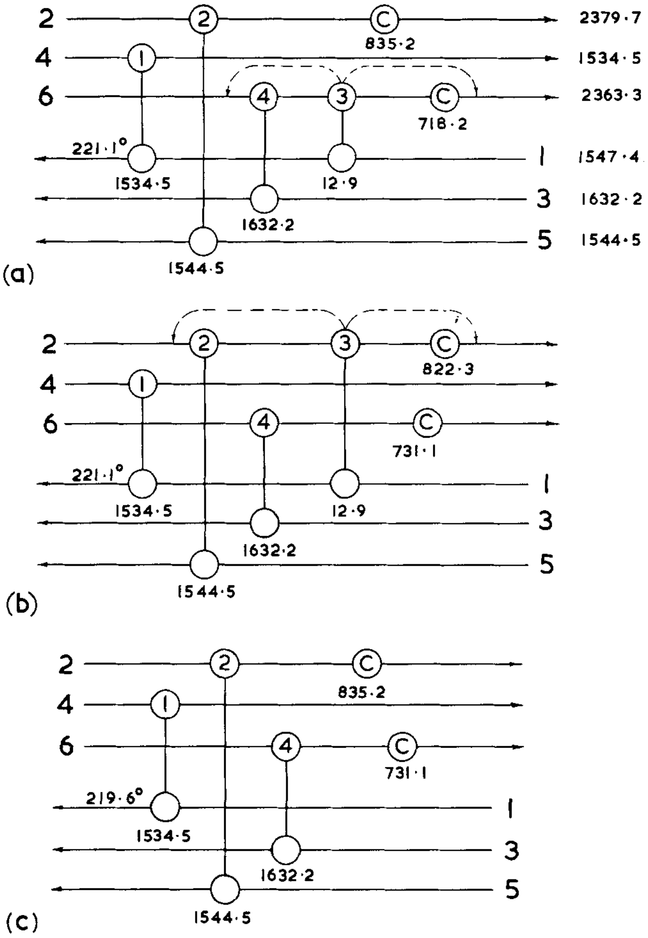


Figure 6. Three solutions for Problem 6SP1.

only six possibilities of introducing the required last two matches: Either streams No. 6 and No. 1 are matched, leaving three possible matches for stream No. 3, or streams No. 6 and No. 3 are matched, leaving three possible matches for stream No. 1. In Table 1, these six possibilities are listed. Possibility No. 5, however, is readily seen to lead to a cyclic topology not feasible in a minimum number of units solution (Hohmann 1971), and possibility No. 6 is seen to be identical with No. 3.

Thus, only the first four possibilities listed in Table 1 have to be evaluated in any detail. This has been done in Figure 5. Two of the four sets were readily seen to be not feasible, in Figure 5a because the heat load of stream No. 2 is not large enough to match both stream No. 3 and stream No. 5, and in Figure 5b because the heat load of stream No. 4 is not large enough to match that of stream No. 3. The remaining two sets, then, are the only ones feasible on grounds of topology, target temperature and heat load compatibility. The heat loads of the individual matches in these selections are shown in Figure 5c and Figure 5d.

Happily, both these selections represent networks that are feasible on grounds of intermediate temperatures as well. In Figure 6, the two networks (6a and 6b) are based on these selections. In both networks, match No. 3 could be placed in three different positions, the alternative positions indicated by means of dashed arrows. Thus the following firm conclusion can now be drawn: There are no more and no less than six feasible networks that solve problem 6SP1 by achieving maximum energy recovery with the minimum number of units, use no split streams, and observe the desired design constraint of $\Delta T_{\min} = 11.11°C$.

Among these six solutions, the two shown in Figure 6 are the two cheapest, costing \$35,017/yr (6a) and \$35,010/yr (6b). Accepting the premise that optimum cost networks are to be found among maximum energy recovery/minimum number of units networks, one may rest assured that structure 6b, first presented by Hohmann in 1971, is indeed the definite optimum cost solution, given the design constraints and costing parameters usually taken (see Pho and Lapidus 1973). In Figure 6c, however, a "practical" solution results from any of the six "exact" minimum number of units solutions, if the exceedingly small match No. 3 is simply left out. This solution would, probably, be considered to be the most promising design in a practical environment. This solution was also noted by Hohmann (1971).

PROBLEM 5SP1

Apparently, there is scope for intelligent and ad hoc formulated thermodynamic arguments when the TC method is employed by hand. A short description of the solution of problem 5SP1 (see Pho and Lapidus 1973) may shed more light on the various kinds of situations that might be encountered. In contrast to the case of 6SP1, there is only one obvious argument for 5SP1 that can be invoked *before* the combinatorial problem is tackled and the main effort has, therefore, to be devoted to finding suitable arguments for the rapid checks of heat lead compatibilities.

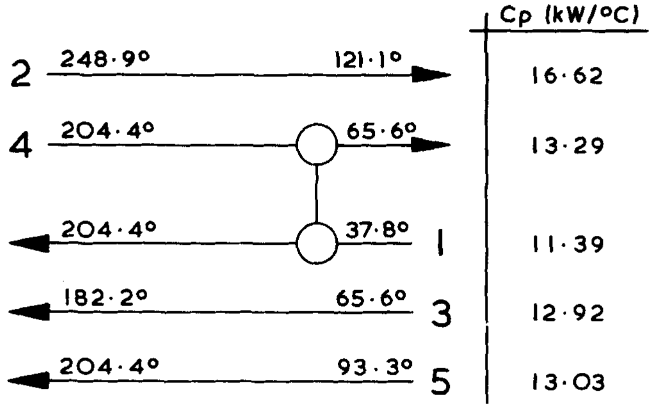


Figure 7. The compulsory match for Problem 5SP1.

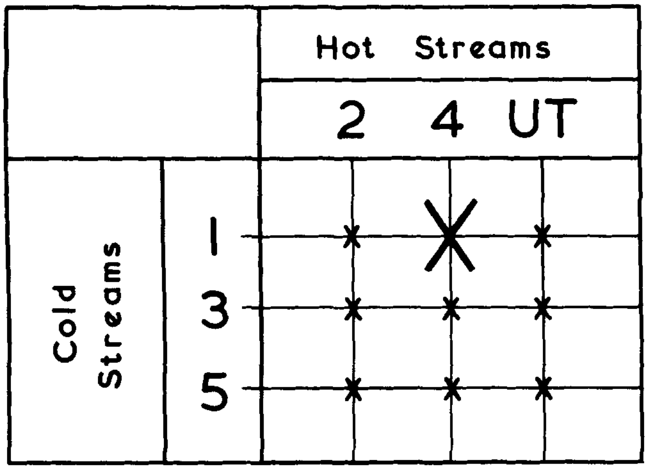


Figure 8. The compulsory and eight more feasible matches for Problem 5SP1.

The utility needs for 5SP1 are 885.1 kW (302×10^4 Btu/hr) of heating and no cooling (Linnhoff and Flower 1978). The minimum number of units is five. With no coolers and the "cold end" stream temperatures as given, a match between streams No. 4 and No. 1 must be specified (Figure 7). The problem remaining is to examine the possibilities that exist for selecting four matches out of eight ones possible (see Figure 8).

The number of such possibilities ($C_8{}^4 = 70$) is still quite large, by the standards of hand calculation, but it is obviously possible to reduce this figure by disregarding all those possibilities in which the utility or process streams remain unconnected. There must be

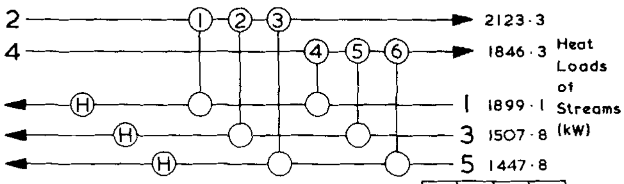
$$C_5{}^4 = 5$$

possibilities in which the utility would remain unconnected, five more in which stream No. 2 would remain unconnected, and five, also, in which either stream No. 3 or stream No. 5 would not be connected. Thus, the total number of possibilities to be considered can be reduced to 50

$$70 - (4 \times 5) = 50$$

Next, the obvious argument may be employed that the stream with the largest head load requires at least two connections. In the case of 5SP1, this leads us to conclude that possibilities with only one match for stream No. 2 are not feasible. Among the first 70 possibilities, 30 ($3 \times C_5{}^3 = 30$) such cases were included. It can be seen that two such possibilities leave stream No. 3 unconnected and two others leave stream No. 5 unconnected. Thus, the total number of possibilities to be considered is finally reduced to

$$50 - (30 - 2 - 2) = 24$$



										(1)	(2)	(3)	
1	X			X	X	X	X			N			
2		X		X	X	X	X			N			
3			X	X	X	X	X			N			
4	X		X	X	X		X			N		N	
5		X	X	X	X		X			N		N	
6			X	X	X		X	X				N	
7			X	X	X		X		X				✓
8	X			X	X		X		X				✓
9		X		X	X		X		X				✓
10	X	X		X		X	X			N		N	
11		X	X	X		X	X			N		N	
12		X		X		X	X	X					✓
13		X		X		X	X		X			N	
14	X			X		X	X	X					✓
15			X	X		X	X	X					✓
16	X	X			X	X	X			N			
17	X		X		X	X	X			N			
18	X				X	X	X	X					✓
19	X				X	X	X		X				✓
20		X	X		X	X	X				N		
21		X			X	X	X	X			N		
22		X			X	X	X		X		N		
23			X		X	X	X	X			N		
24			X		X	X	X		X		N		

Figure 9. The possible sets of matches for Problem 5SP1.

A combinatorial problem of this size can be tackled by hand, and it has been found convenient to use a sketch as shown in Figure 9. In this sketch, the nine possible matches evident from Figure 8 are shown in the network context. Below the diagram, the 24 selections of possible matches identified above are produced systematically, by choosing the connections to stream No. 2 first, and then selecting heaters. A cross under a match indicates its presence in a particular selection.

The first argument used to eliminate selections is based on temperature considerations. Since the heat capacity flowrate of stream No. 1 is less than that of stream No. 4 (11.39 kW/°C versus 13.29 kW/°C), the temperature difference in match No. 4 will decrease from the value 27.8°C at the cold end to a smaller value at the hot end. If the minimum acceptable value is taken as 11.11°C, simple calculation shows that the maximum heat load of match No. 4 is 1,329.4 kW. So match No. 4 cannot, on its own, satisfy the load on stream No. 4, which is 1,846.3 kW. None of the possibilities which feature match No. 4 as the only connection for stream No. 4 need be considered further. These are marked in column (1) in Figure 9.

The other two arguments used are based on more heat load comparisons and are very simple. Unless matched against stream No. 2, stream No. 1 must have at least two connections (see column 2), and any of the cold streams, if connected to a heater, must have at least one further connection to a hot stream (see column 3).

The argument based on the heat load of match No. 4 would not only eliminate sets with match No. 4 as the only connection to stream No. 4, but also all sets with match No. 4 as the only connection to stream No. 1.

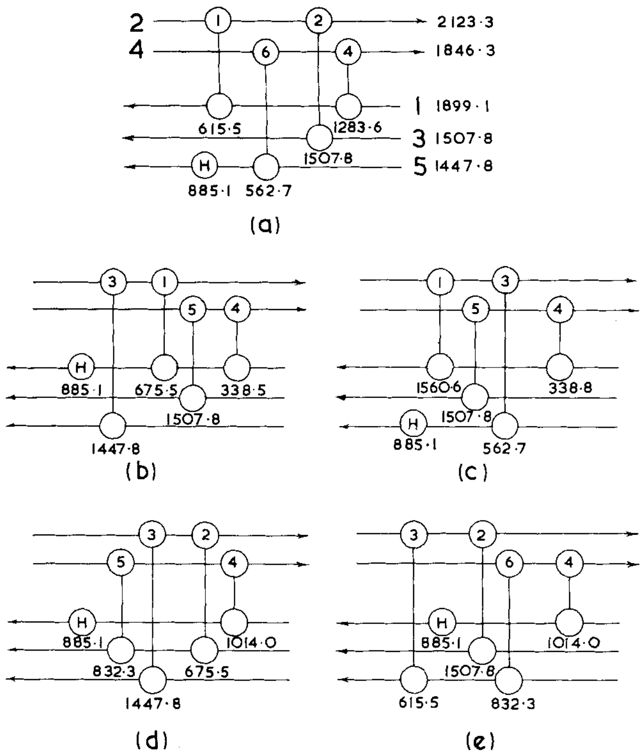
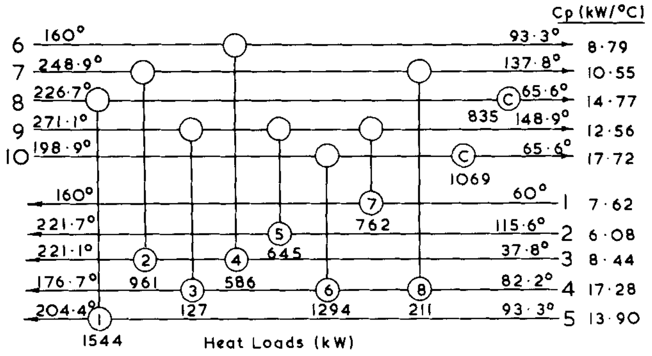
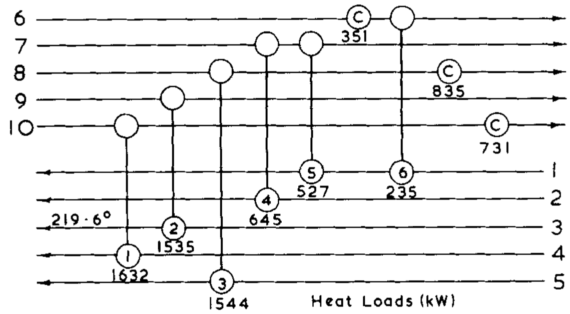


Figure 10. The five solutions of Problem 5SP1.



(a) Optimum cost solution: 43934 $ p.a.



(b) Practical solution with only nine units: 44445 $ p.a.

Figure 11. Two solutions for Problem 10SP1.

This eliminates sets 20 through 24, i.e. duplicates the effect of the argument employed in column (2) in Figure 9. The latter argument has, however, been employed as well, since it does not depend upon a value of $\Delta T_{\min}$. This will be of advantage when the relaxation of the $\Delta T_{\min}$ constraint is discussed.

At this stage, eight possible selections are left. These have been examined and all found feasible, from the point of view of heat loads. In only five cases, however, could networks be found in which the required minimum approach temperature $\Delta T_{\min} = 11.11°C$ (20°F) would be observed. These networks are documented in Figure 10. Selection No. 7 in Figure 9 leads to the structure shown in Figure 10a, selection No. 14 to structure 10b, No. 15 to structure 10c, No. 18 to structure 10d, and No. 19 to structure 10e. In all five cases, the approach temperatures in the individual matches are such that no alternative sequence of matches on any particular stream may be adopted. Thus, one may conclude that there are no more and no less than five networks that solve problem 5SP1, achieve maximum energy recovery, use the minimum number of units with no split streams, and observe the desired design constraint of $\Delta T_{\min} = 11.11°C$.

In Table 2, the annual costs of these networks are given. Solution 10b is the same one as presented by many previous workers as the optimum cost network (without stream splitting). It can now be considered

TABLE 2. ANNUAL COSTS FOR SOLUTIONS FOR 5SP1

Structure	Annual cost ($)
10a	38,336
10b	38,268
10c	38,519
10d	38,550
10e	38,278

to be definitely confirmed as such. However, it is worth emphasizing that a number of alternative solutions can be found if the $\Delta T_{\min}$ constraint is relaxed. Assuming that this constraint was redefined as

$$\Delta T_{\min} = 0$$

the argument based on the approach temperature in match No. 4 in Figure 9 must be reformulated. One then finds that the selection of matches marked in column (1) of Figure 9, are no longer to be rejected.

Finally, feasible networks would emerge from as many as 11 different possibilities shown in Figure 9 (namely from No. 2, No. 3, No. 7, No. 9, No. 12 and Nos. 14—19), and the number of solutions found would be even greater than that, since in most cases, more than one feasible sequence would emerge. Conceivably, one or more of the networks so identified would be cheaper than solution (10b). Thus, it is evident that the design constraint of $\Delta T_{\min} = 20°F$ (111°C) has no other effect in problem 5SP1 than to prevent some feasible and perhaps attractive structures from being identified. This is an important observation which will be taken up later.

THE MERITS OF THE NEW METHOD

Based on the limited evidence gained, it might be suggested that the TC method represents a more promising approach to heat exchanger network synthesis than any other combinatorial method presented to date. It is based on combinatorial principles quite different from tree-searching methods or branch-and-bound algorithms and it appears to be the first method to enable exhaustive searches with no more than a reasonable, if not small, computational effort. The number of enumerations required seems to be less than in conventional combinatorial methods where the problem size is reduced by heuristics, even though no claims for exhaustiveness can be made with these latter methods. For example, solving problem 6SP1 required tackling 132 network problems in the work by Lee, Masso and Rudd (1970).

In the context of hand calculations, the merits of the TC method are more difficult to assess, since the time required to solve a problem will depend on its individual characteristics and on the skill and experience of the user. The demands made on the user appear, however, to be reasonable. This could be important because use of the method by hand will lead to practice in applying thermodynamic and topological arguments of the kind described and will thus improve a user's general design skill.

THE PRINCIPLE OF NON-INDEPENDENT DESIGN OF UNIT OPERATIONS

Most synthesis work on heat exchanger networks to date is based on the heuristic of making every exchanger as large as possible, given the two streams matched. Further, this heuristic seemed to be theoretically justified by Hohmann's (1971) statement that, in a minimum number of units network, each match would terminate at least one stream. However, Hohmann's statement was shown to be fallible and as an example of a network that could, by necessity, not be found using the heuristic in question an almost optimal solution for 10SP1 (Linnhoff and Flower 1978) can be quoted (see match No. 6 in Figure 11a).

A look at the way in which matches are terminated in the TC method allows one to understand this point even better. In the TC method, each match is sized as dictated by the network topology, so as to be compatible with all other matches in the network. Thus, matches may result that are terminated for no obvious reason as far as the two matched streams are concerned. In other words, each match is sized not just from the point of view of the two streams, but with the overall network in mind. This interpretation would appear to expose a vulnerability in conventional synthesis methods, in that they solve network problems by sequential (and non-iterative!) design of unit operations. The remainder of a network being is ignored while a unit operation is designed. The principle of what might be called the "non-independent design of unit operations" is nicely illustrated in this discussion and, quite likely, the implementation of this principle might be a major key for successful future work on systematic design of process networks.

APPROXIMATE SOLUTIONS

Even in the context of a fully automated algorithm, the TC method can help to overcome the unrealistic influence of rigidly fixed process stream temperatures (and heat loads). Any "practical" solution of the kind presented in Figure 6c for 6SP1 or in Figure 11b for 10SP1 has corresponding exact minimum number of units solutions with a very small match. These exact solutions must appear in the exhaustive list of solutions produced by the TC-method and the chance of identifying the practical solution cannot pass unnoticed. It would, however, be necessary to modify the whole procedure in such a way that the thermodynamic and topological arguments used to eliminate rudimentary structures at an early stage are not based on feasibility criteria that are only narrowly missed. (Compare to this the argument used to postulate a second connection).

This opens up a strategy of identifying preferred structures early in the synthesis. It would obviously be possible, once the sets of matches are identified, to further examine those first that feature the smallest matches. Considerable time can be saved by early identification of preferred approximate solutions with less than the minimum number of units, if such solutions exist. In an automated algorithm for industrial applications, such a facility might prove essential when large problems are tackled.

This discussion illustrates another general point. The principle of promoting solutions that are approximate but show some explicit advantage in return is highly desirable in the context of synthesis. Although it might seem unlikely that this principle can be applied easily in an automated algorithm, the above discussion demonstrates that there are instances where this is possible and where it actually may help to reduce the problem size. It appears that the implementation of this principle is another main key for successful future work.

PROBLEM TYPE AND SIZE

If a problem has a difficult heat recovery situation so that maximum energy recovery solutions would only be possible with parallel stream splitting and/or more than the minimum number of units, the TC method as presented above would identify no solution. However, it is precisely in the case of such problems that synthesis by the TI-method is rapid (see Linnhoff and Flower 1978). Namely, the TI method leaves little choice in subnetwork design where heat recovery is difficult, and offers alternative designs in subnetworks only when heat recovery is easier. Thus, a general approach would seem to follow of synthesizing partial networks which ensure maximum energy recovery by the TI method with suitably identified remaining problems. These remaining problems can then be tackled with the TC method.

The size of the problems that may be tackled by the TC method (be they original problems or remaining ones in the sense just outlined) is difficult to estimate without further experience. It seems, however, that practical problems with constraints (such as forbidden matches) would be less difficult to solve than unconstrained literature type problems, given the same number of streams. Also, it seems that the speed of an automated algorithm would largely depend upon how successfully thermodynamic and topological arguments of the type described above can be programmed. Clearly, the right balance must be struck between the time saved with such arguments and the time spent carrying out the necessary tests. One conclusion, however, is firm. Namely, that larger problems will be more amenable to complex solution than generally thought and that the synthesis of heat exchanger networks is, on reflection, not as complex an issue as is usually alleged.

ACKNOWLEDGEMENTS

The British Council provided support for B. Linnhoff during the course of this research.

NOTATION

n_h = number of hot process streams in a problem
n_{hs} = number of hot services required
n_c = number of cold process streams in a problem
n_{cs} = number of cold services required
n_{min} = minimum number of matches, defined by Equation (1)

n_{poss} = number of possible matches, defined by Equation (2)
ΔT_{min} = minimum approach temperature in a match (°C)
Cp = heat capacity flowrate (kW/°C)

Note:

All numerical values quoted are based on calculations using original data in Imperial units. Inevitable rounding on conversion to SI units may lead to small discrepancies.

LITERATURE CITED

Donaldson, R. A., "Studies in the Computer Aided Design of Complex Heat Exchange Networks," Ph.D. thesis, University of Edinburgh (1976).

Hohmann, E. C., "Optimum Networks for Heat Exchange," Ph.D. thesis, University of S. Calif. (1971).

Hohmann, E. C. and F. J. Lockhart, "Optimum Heat Exchanger Network Synthesis," Paper No. 22a, AIChE National Meeting, Atlantic City (1976).

Lee, K. F., A. H. Masso, and D. F. Rudd, "Branch and Bound Synthesis of Integrated Process Designs," *Ind. Eng. Chem. Fundam.*, **9**, 48 (1970).

Linnhoff, B. and J. R. Flower, "Synthesis of Heat Exchanger Networks, I. Systematic Generation of Energy Optimal Networks, and II. Evolutionary Generation of Networks with Various Criteria of Optimality," *AIChE J.*, **24**, 633 (1978).

Linnhoff, B., "Thermodynamic Analysis in the Design of Process Networks," Ph.D. thesis, Univ. of Leeds (1979).

Nishida, N., Y. A. Liu, and L. Lapidus, "Studies in Chemical Process Design and Synthesis: III. A Simple and Practical Approach to the Optimal Synthesis of Heat Exchanger Networks," *AIChE J.*, **23**, 77 (1977).

Pho, T. and L. Lapidus, "Synthesis of Optimal Heat Exchanger Networks by Tree Searching Algorithms," *AIChE J.*, **19**, 1182 (1973).

Ponton, J. W. and R. A. Donaldson, "A Fast Method for the Synthesis of Optimal Heat Exchanger Networks," *Chem. Eng. Sci.*, **29**, 2375 (1974).

Manuscript received February 27, 1978; revision received May 11, 1979 and accepted July 23, 1979.

Axial Dispersion Through Tube Constrictions

Tracer gas dispersion was measured for laminar flow through three different tube constrictions, at orifice Reynolds numbers between 10 and 5000 and Schmidt numbers of 0.213 and 0.769. Each constriction generates a confined jet which significantly enhances axial dispersion at intermediate Reynolds numbers ranging from 100 to 1000.

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SCOPE

In chronic lung diseases, such as bronchitis and asthma, there may be a pathological narrowing of airways due to abnormal mucous secretion, tissue inflammation, or a loss of tissue rigidity. Such a constriction may affect the transport of respired gases, both by its influence upon bulk flow and upon longitudinal mixing. In previous work we demonstrated how the dispersion of a tracer gas pulse is a convenient measurement of mixing in physical tube network models (Ultman and Blatman 1977) as well as in the human lung (Ultman et al. 1978). The object of the current work is to evaluate the effect of a constriction upon tracer gas dispersion in a single airway.

We employed three alternative models of an airway constriction, two sharp-edged orifices with hole-to-tube diameter ratios (β) of 0.2 and 0.5 and a tapering constriction with β = 0.33. Air flows corresponding to orifice Reynolds numbers (Re_o) from 10 to 5000 are investigated, with both helium (He) and pure oxygen (O_2) tracer gases. The results are expressed as the excess dispersion induced by the constriction relative to dispersion in the unconstricted tube.

CONCLUSIONS AND SIGNIFICANCE

In general, a tube constriction causes significant excess dispersion when $Sc < 1$ and when Re_o is in the range of 100 to 1000; below a critical orifice Reynolds number of approximately 600, the excess dispersion is an increasing function of Re_o. Above the critical value, it is a sharply decreasing function of Re_o. A Taylor-type analysis of the data verifies that the excess dispersion is caused by radial diffusion of tracer from a downstream orifice jet into the slowly circulating stall which it creates.

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